Numerical Study of 3D Incompressible Euler Equations with Highly Symmetric Initial Data

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Abstract:

The Kida-Pelz type high symmetric initial value for 3D incompressible flow is a very attractive candidate as the invisible flow with possible finite time blowup behavior. The numerical results for the Navier-Stokes equations indicate strongly that there will be a finite time blowup. In 1997, Pelz presented a filament model to mimic this blowup behavior observed in the Navier-Stokes computation and a finite time blowup with certain scaling properties was produced. To check if the blowup of the filament model is due to the modelling error of the filament model equation, or due to the nature of the Euler equations, we revised this filament model to a periodic configuration, which can be numerical solved by both the filament model equation and the full 3D Euler solver. The periodic filament model equation was numerical solved with an Ewald summation technique. The blowup behavior with the same scaling properties reported by Pelz was observed again in our computation, which appeared quite robust to the perturbation on the profile of the initial filament. The full Euler equations were solved with a standard pseudo-spectral code with a recently developed high order Fourier filter to remove the aliasing error. To our surprise, the numerical results of the full Euler equations deviated from the numerical results of the filament model simulation and behaved as a comparative stable structure. Therefore, the most possibility based on current numerical results is that the high symmetric filament model missed certain intrinsic properties in the Euler equation to produce the strange numerical behavior observed in the Navier-Stokes computation.